Principles of Intelligent Systems: First-Order Logic*

Lecture 10

*These slides are based on Chapter 8 of Russell and Norvig's *Artificial Intelligence: A modern approach* (http://aima.eecs.berkeley.edu/slides-pdf/)

Problems with propositional logic

From the last lecture: propositional logic lacks the expressive power to describe a complex environment *concisely*

For example we needed a separate rule for breezes and pits for each square:

$$B_{1,1} \quad \Leftrightarrow \quad (P_{1,2} \vee P_{2,1})$$

Whereas we really wanted to say: Squares adjacent to pits are breezy

We need something more like natural language to express problems concisely but we need a context-independent, unambiguous language with which we can reason using computational methods

Answer: First-order logic

Propositional versus first-order logic

Propositional logic assumes the world is up of facts
that are either true or false

First-order logic assumes more: the world is made up of objects
that can have relations between them that either do or do not hold

First-order logic can express relations between objects, such as one wumpus square (object) being adjacent to another, or properties of an object, such as a square being breezy.

In addition first-order logic can express functions:
a function is a relation in which there is only one value for any given input

For example, "brother of" is a relation between two objects whereas "father of" is a function, because (biologically) a person can only have one father, but can have many brothers

Symbols

First-order logic has three basic syntactic elements:

Constant symbols which stand for objects

Predicate symbols which stand for relations

Function symbols which stand for functions

In the example from the book, modelling the world of King Richard and King John, there are:

the constant symbols *John* and *Richard* (representing them as objects)
the predicate symbols *Brother*, *OnHead*, *Person*, *King* and *Crown*
the function symbol *LeftLeg*
(because it is assumed an object can have only one left leg)

Semantics and Interpretations

Semantics relate sentences to models in order to determine truth.

For this we need an interpretation that specifies exactly which objects, relations and functions are referred to by the constant, predicate and function symbols.

The intended interpretation of the world of John and Richard is:

– *Richard* refers to Richard the Lionheart and *John* to good King John

– *Brother* refers to the brotherhood relation (John is the brother of Richard *and* Richard is the brother of John); *OnHead* refers to the "on head" relation that holds between the crown and King John; *Person, King* and *Crown* refer to the sets of objects that are persons kings and crowns.

– *LeftLeg* refers to the "left leg" function, that is the mapping of a left leg to Richard and John.

Terms

A term is a logical expression that refers to an object

Hence a constant symbol is a term. However it is often impractical to have a distinct symbol to name every object (for example to name every possible shift a nurse could work in a nurse roster)

This is what function symbols are for: to refer to a particular object in terms of something else

For example $LeftLeg(John)$ refers to John's left leg without giving the leg a name
Or $Shift(Jane, Sunday)$ could refer to the shift Jane is working this Sunday.

The point is a function is simply a complex kind of name.

Atomic sentences

Putting terms (referring to objects) together with predicate symbols (referring to relations) gives an atomic sentence

This is a predicate symbol followed by a parenthesized list of terms, e.g.:

$$Brother(Richard, John).$$

Atomic sentences can also be given complex terms (i.e. functions) as arguments, e.g:

$$Married(Father(Richard), Mother(John)).$$

An atomic sentence is **true** *in a given model, under a given interpretation, if the relation referred to by the predicate symbol holds among the objects referred to by the arguments*

Complex sentences

As with the propositions of propositional logic, we can use logical connectives to construct more complex first-order sentences:

$$\neg Brother(LeftLeg(Richard), John)$$
$$Brother(Richard, John) \wedge Brother(John, Richard)$$
$$King(Richard) \vee King(John)$$
$$\neg King(Richard) \Rightarrow King(John).$$

Universal quantifiers and variables

The main difficulty with propositional logic was the inability to express general rules that apply to collections of objects.

First-order logic allows this by introducing quantifiers and variables

Universal quantification allows us to construct sentences that can say things like "All kings are persons", as follows:

$$\forall x \; King(x) \Rightarrow Person(x).$$

This reads: "For all x, if x is a king, then x is a person"
The symbol x is a **variable**
By convention variables are lowercase letters
A variable is a term in its own right and can be the argument of a function, e.g.: $LeftLeg(x)$
A term with no variables is called a **ground term**, i.e. the interpretation grounds it in an actual object in the world

Existential quantifiers

A universal quantifier allows us to make statements about *all* objects, whereas an existential quantifier allows us to make a statement about *some* object without actually naming it

For example, to say that King John has a crown on his head, we write:

$$\exists x \; Crown(x) \land OnHead(x, John).$$

This reads:
"There exists an x such that x is a crown and x is on John's head"

Quantifiers can be mixed and nested. For example, to say that "Everybody loves somebody" we write:

$$\forall x \; \exists y \; Loves(x, y).$$

Equality

Finally, first-order logic contains the equality symbol which allow statements to the effect that two terms refer to the same object, for instance:

$$Father(John) = Henry$$

This allows the expression of things being not equal as follows:

$$\exists x, y\ Brother(x, Richard) \wedge Brother(y, Richard) \wedge \neg(x = y).$$

The intended meaning of this sentence is that Richard has at least two brothers

Omitting $\neg(x = y)$ would allow $x = y$ and hence both x and y could be assigned to $John$ making the sentence true in the case where Richard only has one brother

Syntax summary

Sentence $\rightarrow$ *AtomicSentence*
$\mid$ (*Sentence Connective Sentence*)
$\mid$ *Quantifier Variable*, ... *Sentence*
$\mid$ $\neg$ *Sentence*

AtomicSentence $\rightarrow$ *Function*(*Term*, ...)

Term $\rightarrow$ *Function*(*Term*, ...)
$\mid$ *Constant*
$\mid$ *Variable*

Connective $\rightarrow$ $\Rightarrow \mid \wedge \mid \vee \mid \Leftrightarrow$
Quantifier $\rightarrow$ $\forall \mid \exists$
Constant $\rightarrow$ $A \mid X_1 \mid$ *John* $\mid \ldots$
Variable $\rightarrow$ *a* $\mid$ *x* $\mid$ *s* $\mid \ldots$
Predicate $\rightarrow$ *Brother* $\mid$ *OnHead* $\mid$ *Person* $\mid \ldots$
Function $\rightarrow$ *LeftLeg* $\mid$ *Father* $\mid \ldots$