Principles of Intelligent Systems: Constraint Satisfaction Problems*

Lecture 6

*These slides are taken from the Chapter 4b slides of Russell and Norvig's *Artificial Intelligence: A modern approach* (http://aima.eecs.berkeley.edu/slides-pdf/)

Outline

◊ Definitions

◊ CSP examples

◊ Backtracking search for CSPs

Constraint satisfaction problems (CSPs)

Standard search problem:

state is a "black box"—any old data structure

that supports goal test, eval, successor

CSP:

state is defined by *variables* X_i with *values* from *domain* D_i

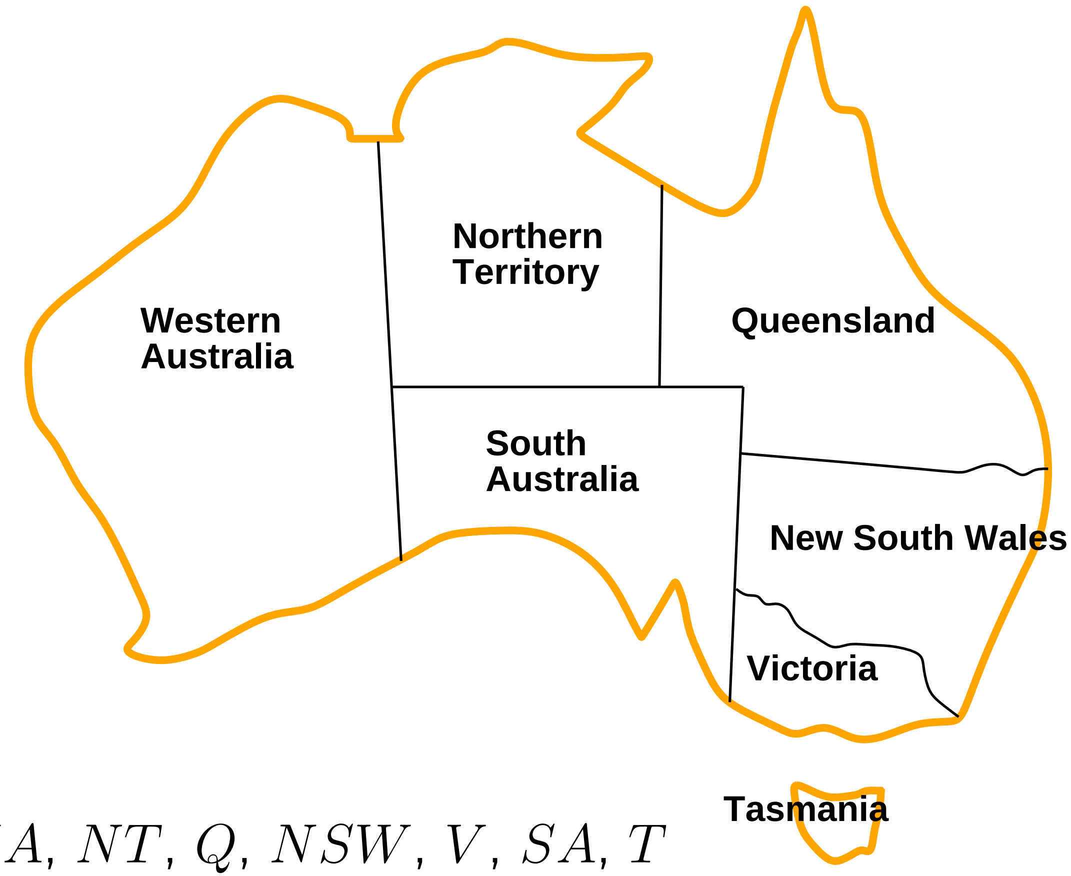
goal test is a set of *constraints* specifying

allowable combinations of values for subsets of variables

Simple example of a *formal representation language*

Allows useful *general-purpose* algorithms with more power than standard search algorithms

Example: Map-Coloring



Variables WA, NT, Q, NSW, V, SA, T

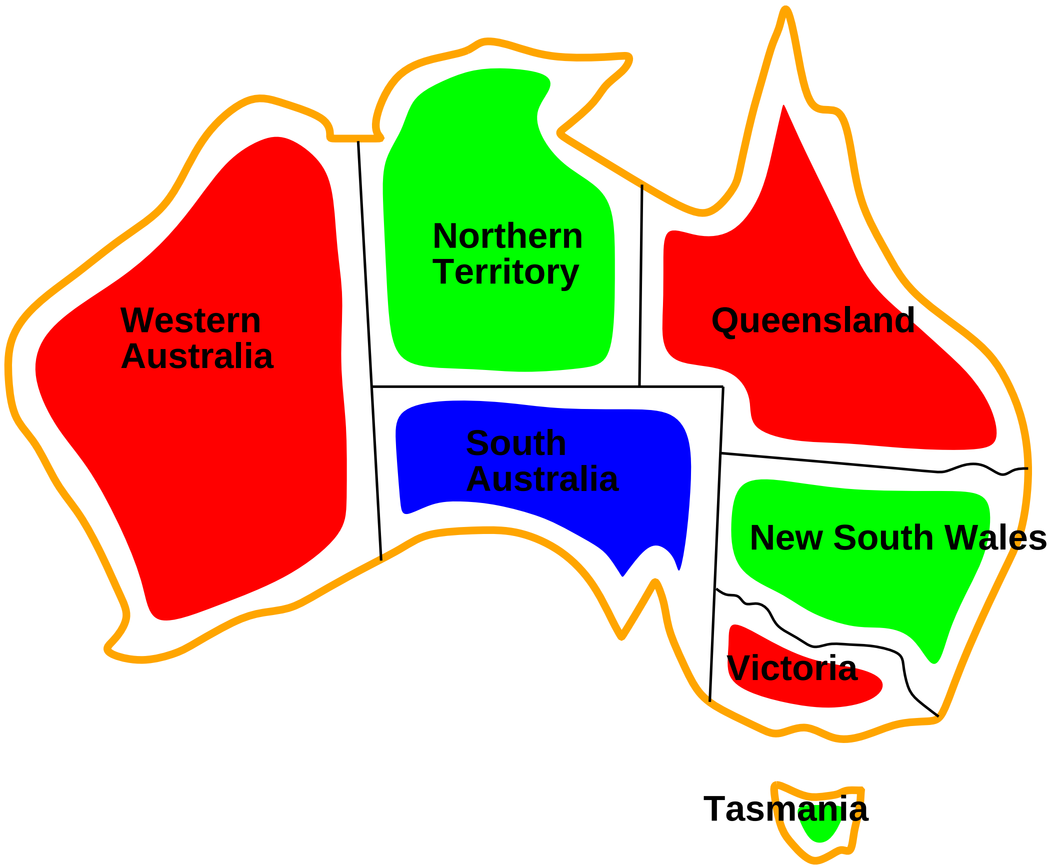
Domains $D_i = \{red, green, blue\}$

Constraints: adjacent regions must have different colors

e.g., $WA \neq NT$ (if the language allows this), or

$(WA, NT) \in \{(red, green), (red, blue), (green, red), (green, blue), \ldots\}$

Example: Map-Coloring contd.

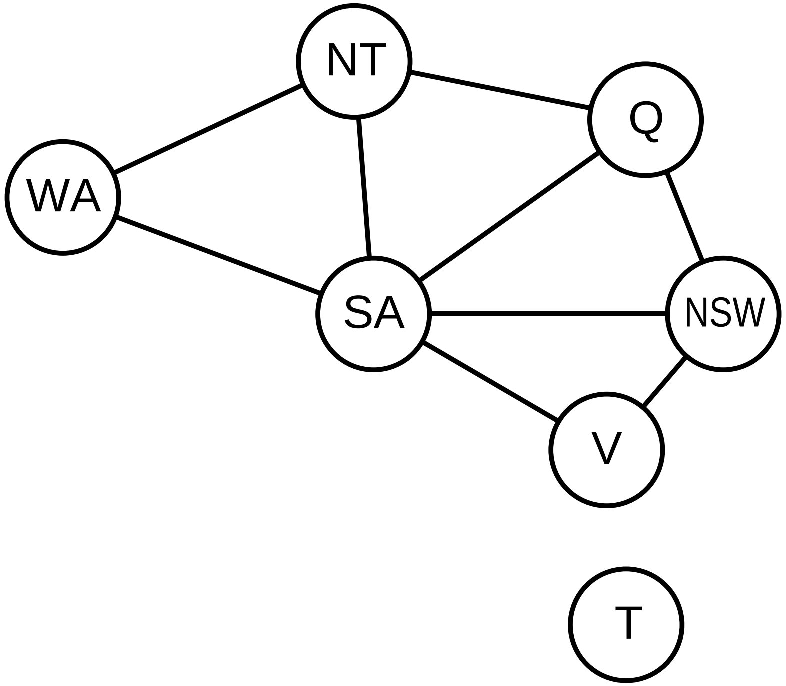


Solutions are assignments satisfying all constraints, e.g.,
$\{WA = red, NT = green, Q = red, NSW = green, V = red, SA = blue, T = green\}$

Constraint graph

Binary CSP: each constraint relates at most two variables

Constraint graph: nodes are variables, arcs show constraints



General-purpose CSP algorithms use the graph structure
to speed up search. E.g., Tasmania is an independent subproblem!

Varieties of CSPs

Discrete variables

finite domains; size $d \Rightarrow O(d^n)$ complete assignments

- ♢ e.g., Boolean CSPs, incl. Boolean satisfiability (NP-complete)

infinite domains (integers, strings, etc.)

- ♢ e.g., job scheduling, variables are start/end days for each job
- ♢ need a constraint language, e.g., $StartJob_1 + 5 \leq StartJob_3$
- ♢ linear constraints solvable, nonlinear undecidable

Continuous variables

- ♢ e.g., start/end times for Hubble Telescope observations
- ♢ linear constraints solvable in poly time by LP methods

Varieties of constraints

Unary constraints involve a single variable,
e.g., $SA \neq green$

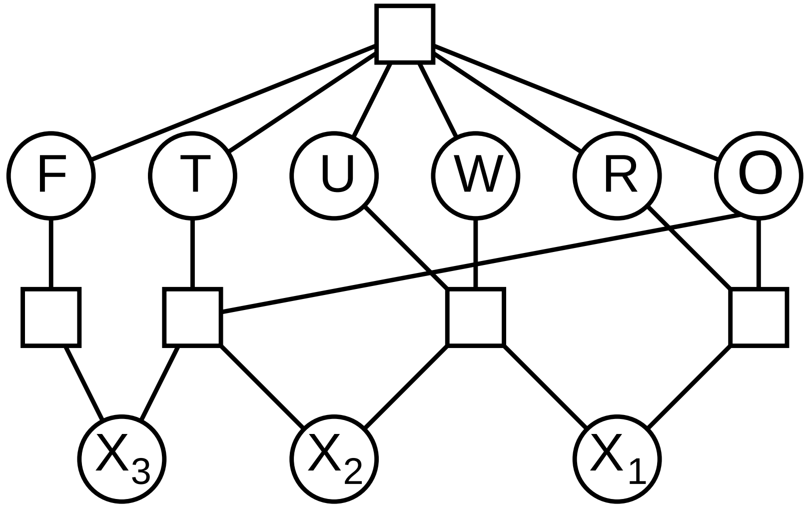
Binary constraints involve pairs of variables,
e.g., $SA \neq WA$

Higher-order constraints involve 3 or more variables,
e.g., cryptarithmetic column constraints

Preferences (soft constraints), e.g., red is better than $green$
often representable by a cost for each variable assignment
$\rightarrow$ constrained optimization problems

Example: Cryptarithmetic

```
    T W O
+   T W O
---------
  F O U R
```



Variables: $F\ T\ U\ W\ R\ O\ X_1\ X_2\ X_3$
Domains: $\{0,1,2,3,4,5,6,7,8,9\}$
Constraints
  alldiff(F,T,U,W,R,O)
  $O+O=R+10\cdot X_1$, etc.

Real-world CSPs

Assignment problems
e.g., who teaches what class

Timetabling problems
e.g., which class is offered when and where?

Hardware configuration

Spreadsheets

Transportation scheduling

Factory scheduling

Floorplanning

Notice that many real-world problems involve real-valued variables

Standard search formulation (incremental)

Let's start with the straightforward, dumb approach, then fix it

States are defined by the values assigned so far

♢ Initial state: the empty assignment, $\{\,\}$

♢ Successor function: assign a value to an unassigned variable
that does not conflict with current assignment.
$\Rightarrow$ fail if no legal assignments (not fixable!)

♢ Goal test: the current assignment is complete

1) This is the same for all CSPs!
2) Every solution appears at depth n with n variables
$\Rightarrow$ use depth-first search
3) Path is irrelevant, so can also use complete-state formulation
4) $b = (n - \ell)d$ at depth ℓ, hence $n!d^n$ leaves!!!!

Backtracking search

Variable assignments are commutative, i.e.,

$[WA = red$ then $NT = green]$ same as $[NT = green$ then $WA = red]$

Only need to consider assignments to a single variable at each node

$\Rightarrow$ $b = d$ and there are d^n leaves

Depth-first search for CSPs with single-variable assignments
is called backtracking search

Backtracking search is the basic uninformed algorithm for CSPs

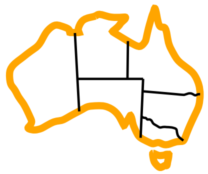
Can solve n-queens for $n \approx 25$

Backtracking search

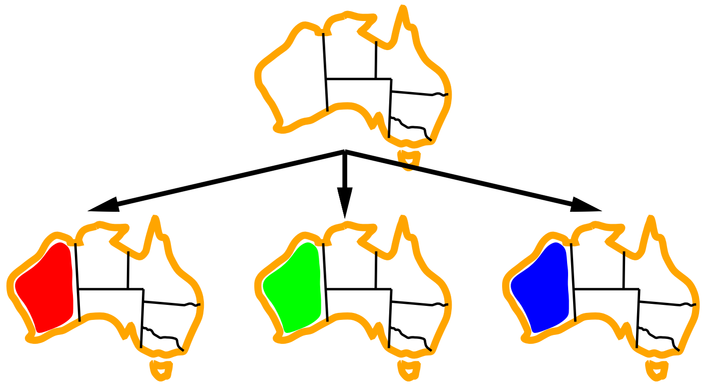
function BACKTRACKING-SEARCH(*csp*) **returns** solution/failure
    return RECURSIVE-BACKTRACKING([ ], *csp*)

function RECURSIVE-BACKTRACKING(*assigned*, *csp*) **returns** solution/failure
    if *assigned* is complete **then return** *assigned*
    $var \leftarrow$ SELECT-UNASSIGNED-VARIABLE(VARIABLES[*csp*], *assigned*, *csp*)
    for each *value* **in** ORDER-DOMAIN-VALUES(*var*, *assigned*, *csp*) **do**
        if *value* is consistent with *assigned* according to CONSTRAINTS[*csp*] **then**
            $result \leftarrow$ RECURSIVE-BACKTRACKING($[var = value | assigned]$, *csp*)
            if $result \neq failure$ **then return** *result*
    end
    return *failure*

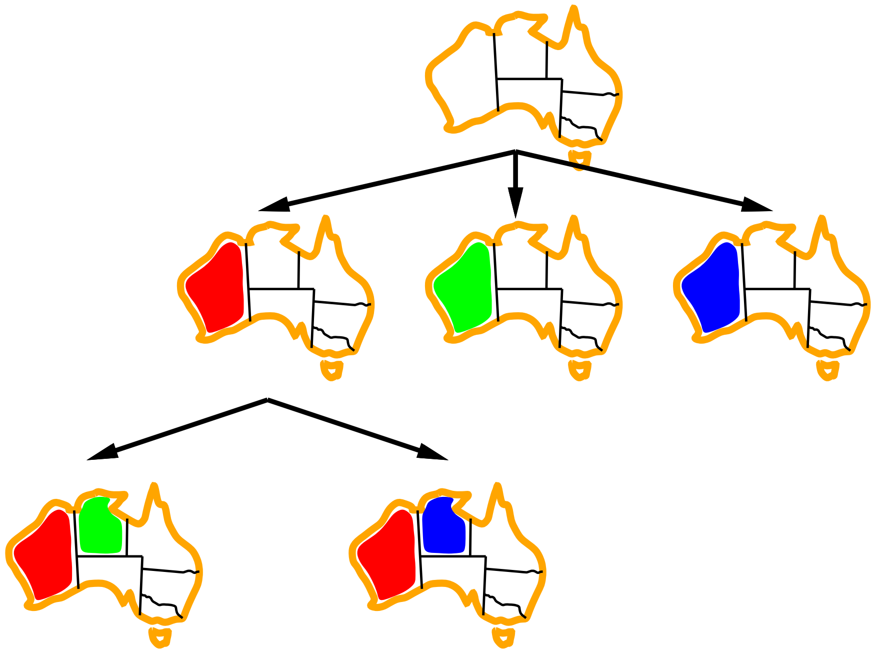
Backtracking example



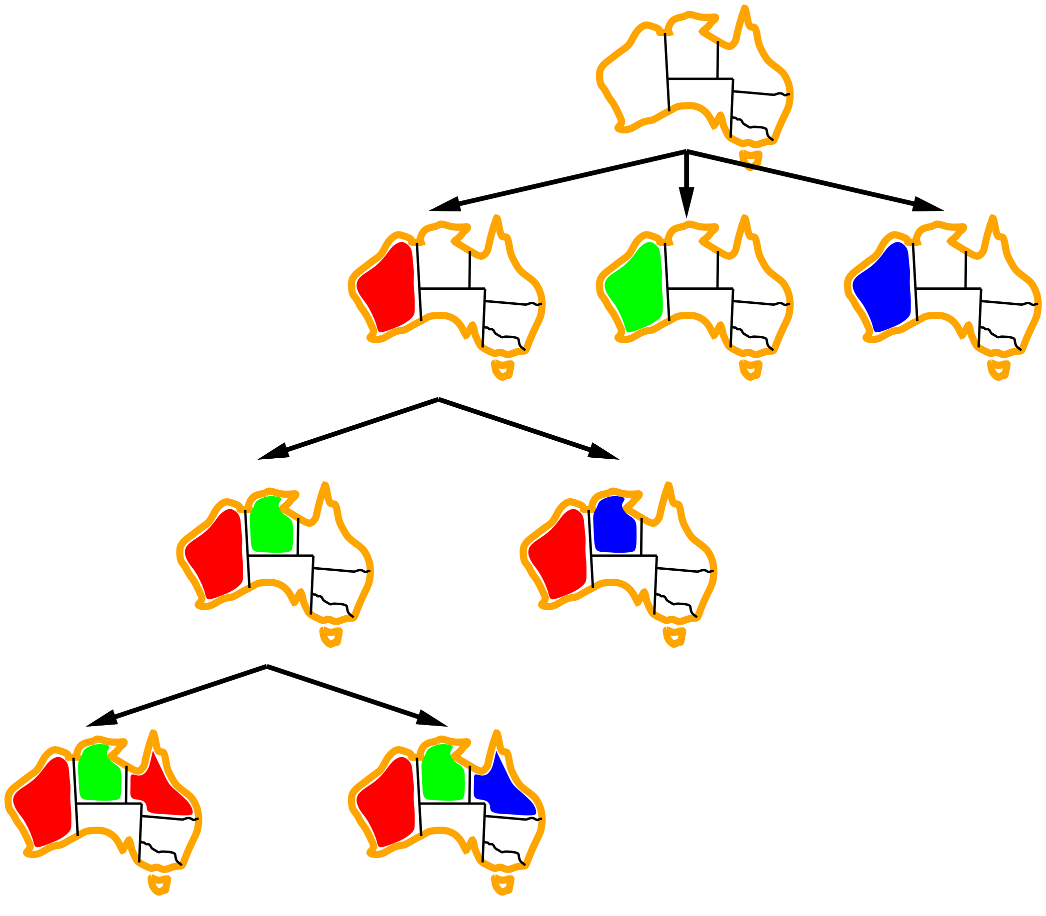
Backtracking example



Backtracking example



Backtracking example



Summary

CSPs are a special kind of problem:
  states defined by values of a fixed set of variables
  goal test defined by *constraints* on variable values

Backtracking = depth-first search with one variable assigned per node